CSE 250
Data Structures

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Day 15
Stacks, Queues, and Mazes (oh my)
Textbook Ch. 7

Recap

Stacks: Last In First Out (LIFO)

- Push (put item on top of the stack)
- Pop (take item off top of stack)
- Top (peek at top of stack)

Queues: First in First Out (FIFO)

- Enqueue (put item on the end of the queue)
- Dequeue (take item off the front of the queue)
- Head (peek at the item in the front of the queue)

Recap

Stacks: Last In First Out (LIFO)

- Push (put item on top of the stack) $\Theta(1)$ (or amortized $O(1)$)
- Pop (take item off top of stack) $\Theta(1)$
- Top (peek at top of stack) $\Theta(1)$

Queues: First in First Out (FIFO)

- Enqueue (put item on the end of the queue) $\Theta(1)$
- Dequeue (take item off the front of the queue) $\Theta(1)$
- Head (peek at the item in the front of the queue) $\Theta(1)$

Queues

Thought Question: How could you use an array to build a queue?

Queues

ArrayBuffer Attempt 1

Enqueue: Append(...)

Dequeue: Remove(0)

Queues

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Dequeue: Remove(0)

What is the complexity?

Queues

ArrayBuffer Attempt 1

Enqueue: Append(...)	**Amortized $O(1)$**
Dequeue: Remove(0)	**$O(n)$**

What is the complexity?

Queues

ArrayBuffer Attempt 2

Enqueue: Insert(0)

Dequeue: Remove(last)

Queues

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Queues

ArrayBuffer Attempt 2

Enqueue: Insert(0) $O(n)$

Dequeue: Remove(last) $\Theta(1)$

What is the complexity?

Queues

Can we avoid the cost of moving all of the elements forward or backward each time we add or remove?

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Why didn't we have to pay that cost with a list?

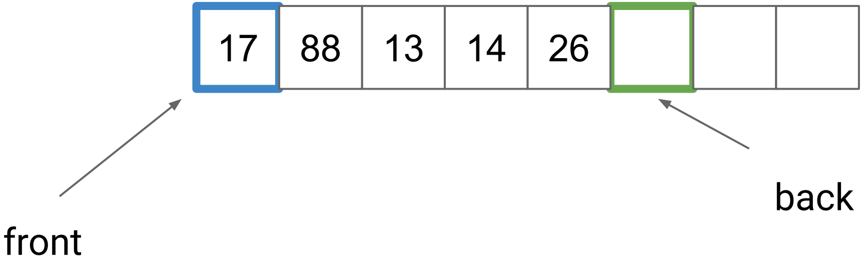
Queues

Can we avoid the cost of moving all of the elements forward or backward each time we add or remove?

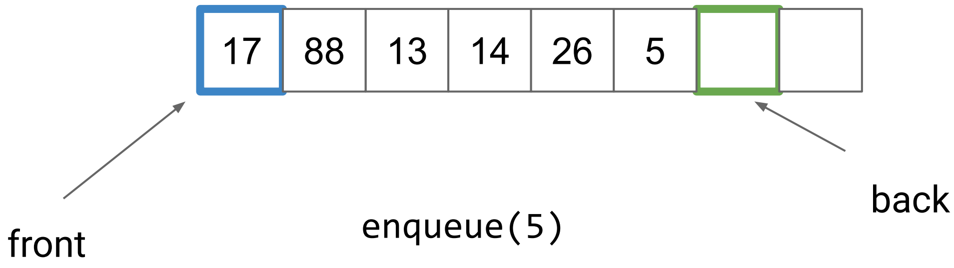
Why didn't we have to pay that cost with a list?

Update our values of "first" and "last"!

Queues



Queues



enqueue(5)

Queues


```
enqueue(5)
enqueue(4)
```

Queues


```
enqueue(5)
enqueue(4)
dequeue()
```

Queues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enqueue(5)
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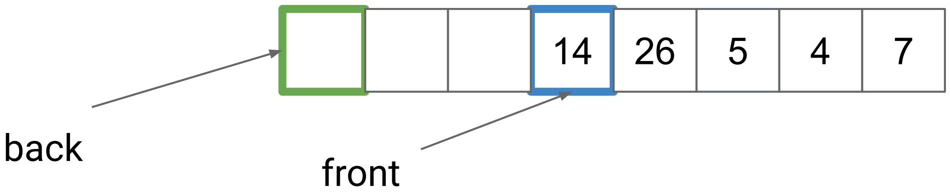
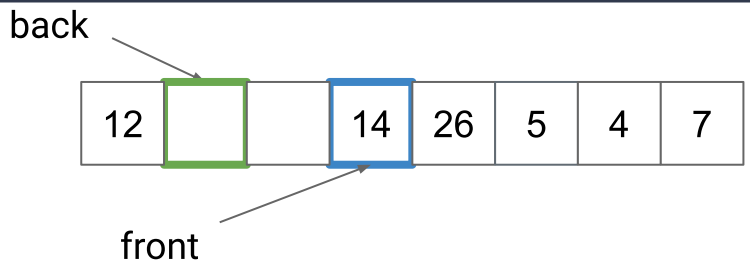


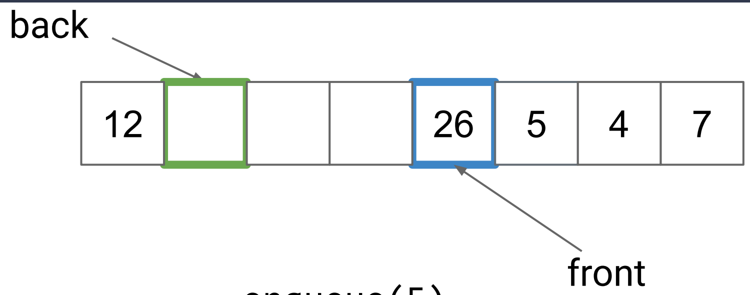

```
enqueue(5)
enqueue(4)
dequeue()
dequeue()
dequeue()
enqueue(7)
```

Queues



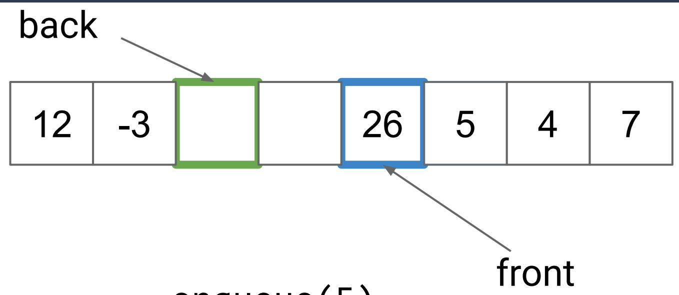

```
enqueue(5)
enqueue(4)
dequeue()
dequeue()
dequeue()
enqueue(7)
enqueue(12)
```

Queues




```
enqueue(5)
enqueue(4)
dequeue()
dequeue()
dequeue()
enqueue(7)
enqueue(12)
dequeue()
```

Queues




```
enqueue(5)
enqueue(4)
dequeue()
dequeue()
dequeue()
enqueue(7)
enqueue(12)
dequeue()
enqueue(-3)
```

ArrayDeque (Resizable Ring Buffer)

Active Array = [start, end)

Enqueue

1. Resize buffer if needed
2. Add new element at buffer[end]
3. Advance end pointer (wrap to front as needed)

Dequeue

1. Remove element at buffer[start]
2. Advance start pointer (wrap to front as needed)

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What is the complexity?

ArrayDeque (Resizable Ring Buffer)

Active Array = [start, end)

Enqueue Amortized $O(1)$

1. Resize buffer if needed
2. Add new element at buffer[end]
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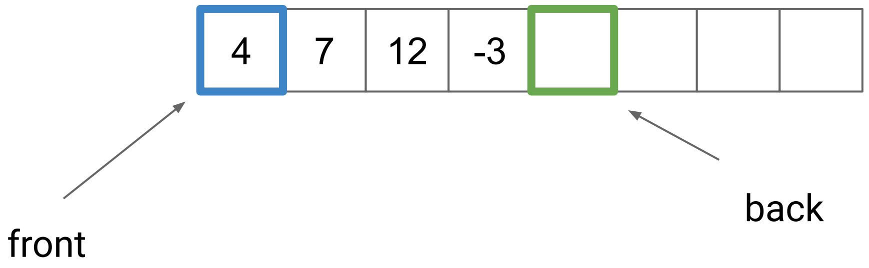
Dequeue $\Theta(1)$

1. Remove element at buffer[start]
2. Advance start pointer (wrap to front as needed)

What is the complexity?

Why Ring Buffer?



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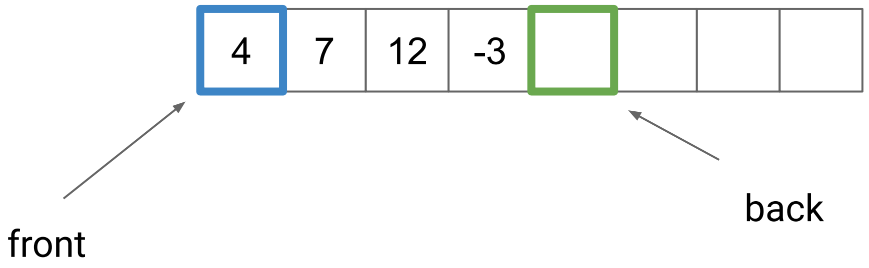
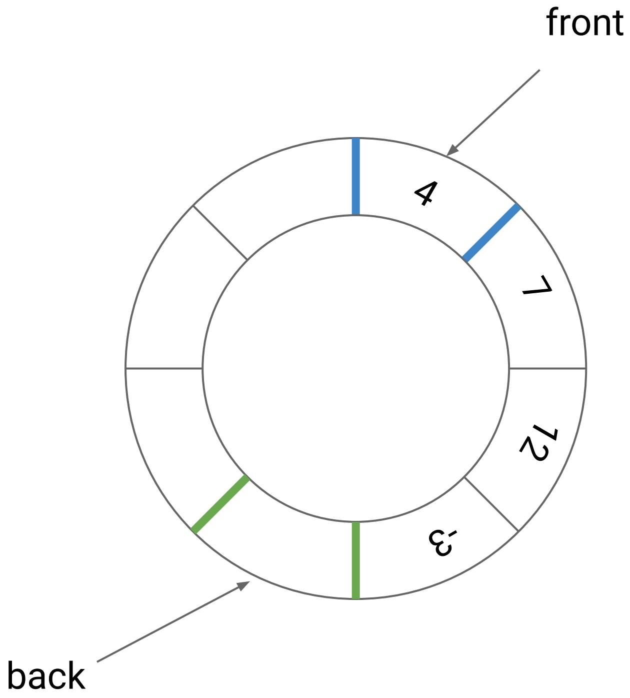


Conceptually, we can think of this as a ring…

Why Ring Buffer?



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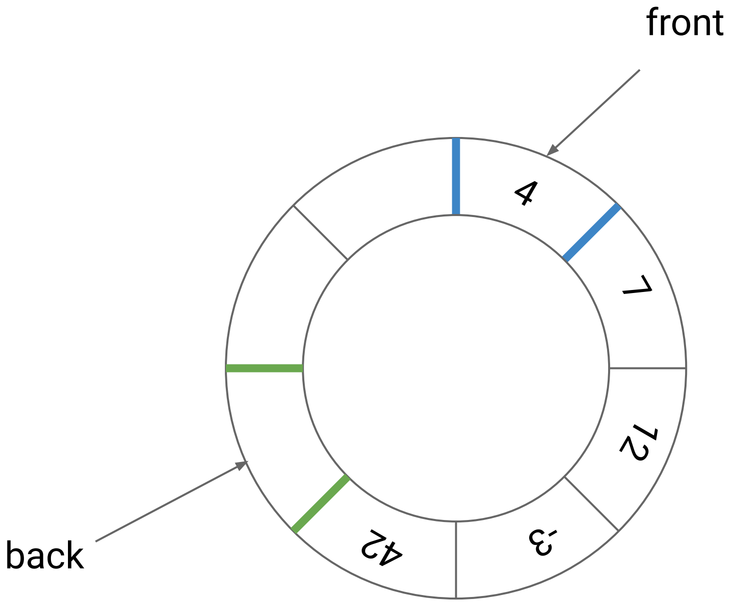
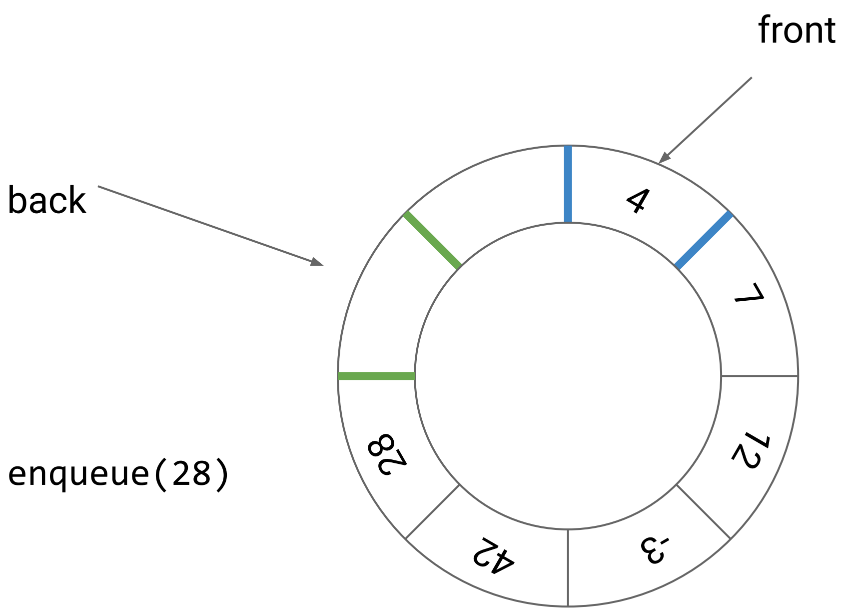


`enqueue(42)`

Why Ring Buffer?



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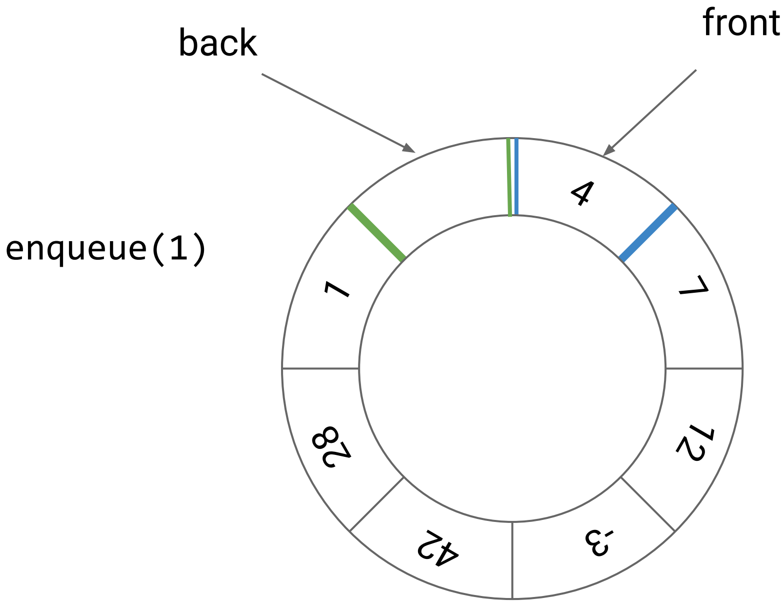

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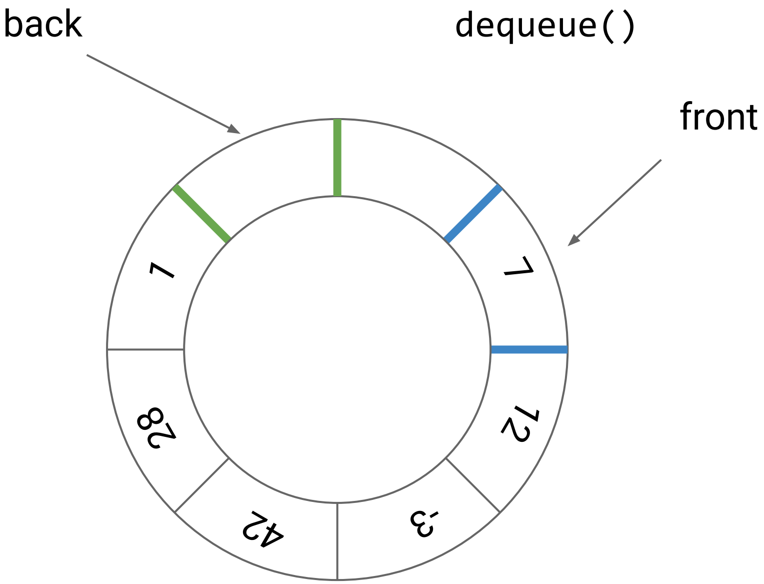

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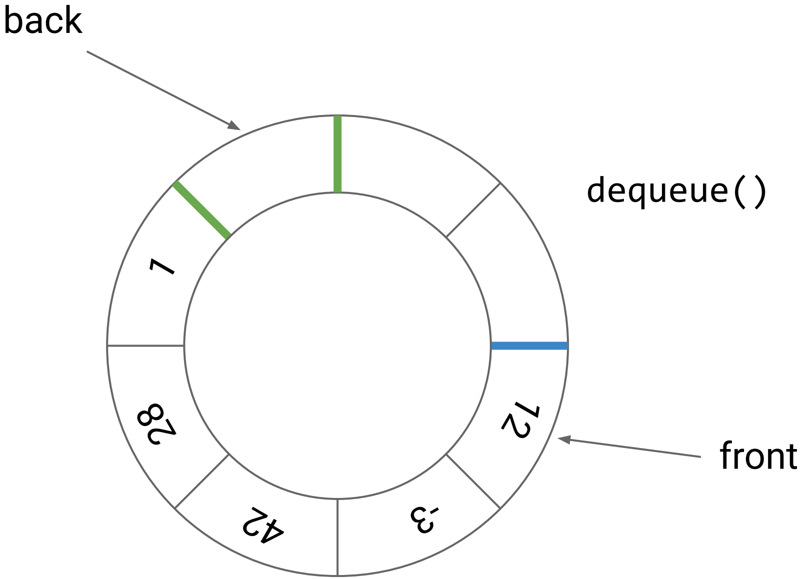

Applications of Stacks and Queue

Stack: Checking for balanced parentheses/braces

Queue: Scheduling packets for delivery

Both: Searching mazes

Balanced Parentheses/Braces

What does it mean for parentheses/braces to be balanced?

1. Every opening symbol is matched by a closing symbol
2. No nesting overlaps (ie `{(})` is not ok).

`{()({})}`   `{())`   `())`

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✓

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`{()({})}`	`{())`	`())`
✓	✗	✗

Idea #1

Idea: Count the number of unmatched open parens/braces.

Increment counter on `(`, decrement on `)`

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Increment counter on `(`, decrement on `)`

Problem: allows for `{(})`

Idea #2

Idea: Track nesting on a stack!

On `(` or `{`, push the symbol on the stack.

On `)` or `}`, pop the stack and check for a match.

Demo in Section B Slides:

[https://odin.cse.buffalo.edu/teaching/cse-250/2022fa/slide/14b-QueueStackApps.html#/13]

Network Packets

Router: 1gb/s internal network, 100mb/s external

- 1 gb/s sent to the router, but only 100mb/s can leave.
- How do we handle this?

Queues

- Enqueue data packets in the order they are received.
- When there is available outgoing bandwidth, dequeue and send.

Avoiding Queueing Delays

- Limit size of queue; Packets that don't fit are dropped

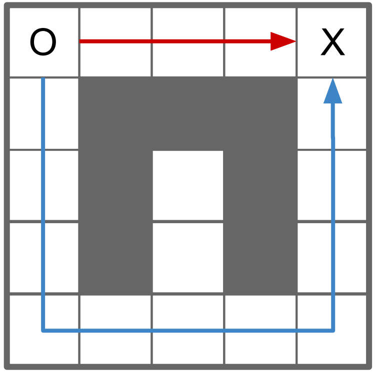
TCP: blocked packets are retried

UDP: application deals with dropped packets

Mazes

O is the start, **X** is the objective

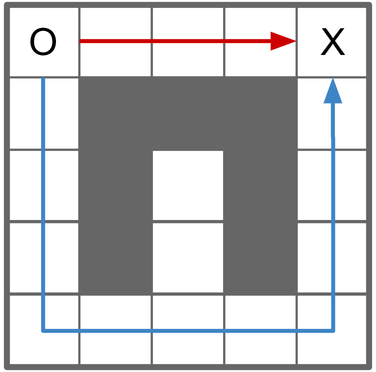
- There may be multiple paths
- Generally, we want the shortest



Approach 1: Take the first available route in one direction

- Right, Down, Left, or Up
- Down, Right, Up, or Left

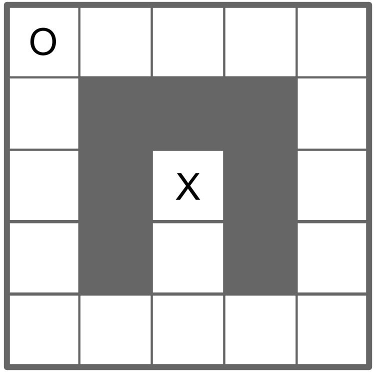
Mazes



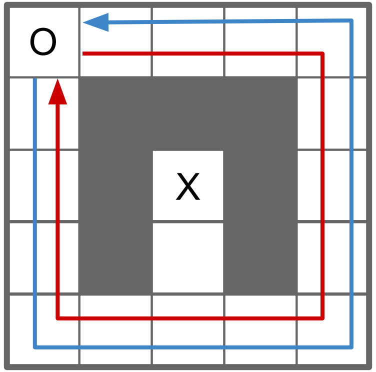
How do you know which one is best?

Is there anything wrong with this algorithm?

Mazes



Mazes



Priority order doesn't guarantee exploring the entire maze

Formalizing Maze-Solving

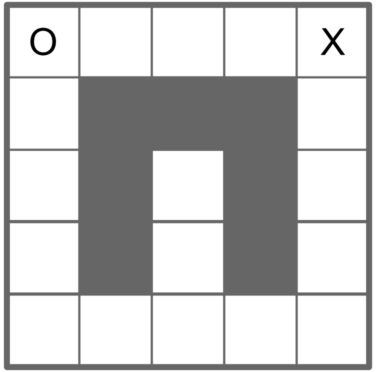
Inputs:

- The map: an n x m grid of squares which are either filled or empty
- The **O** is at position *start*
- The **X** is at position *dest*

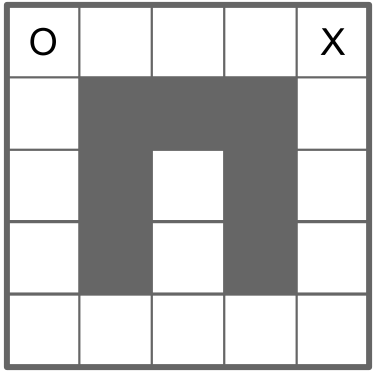
Goal: Compute `steps(start, dest)`, the minimum number of steps from start to end.

How do we define the `steps` *function?*

Mazes

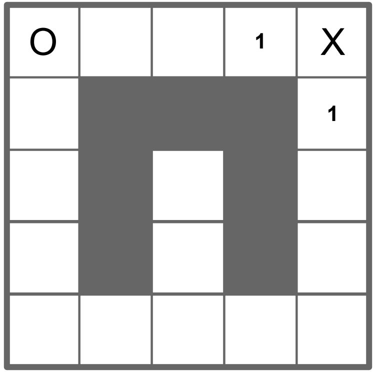


Mazes



How many steps are required for the squares right next to X?

Mazes



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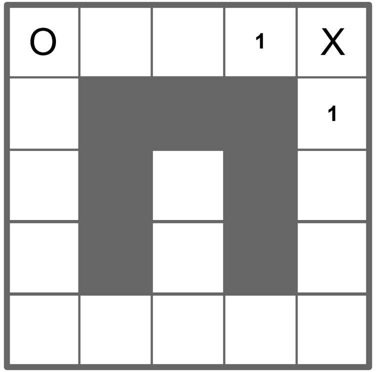
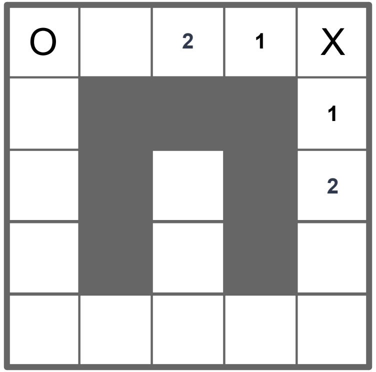


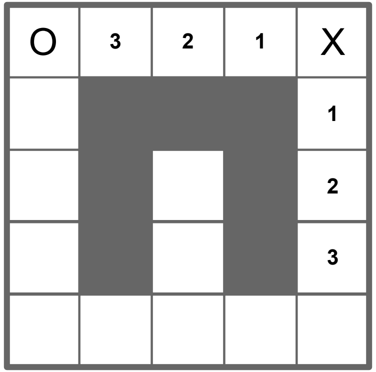
How many steps are required for the squares right next to X?

And the squares next to those?

Mazes



Mazes



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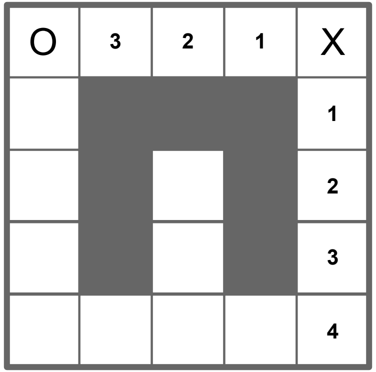
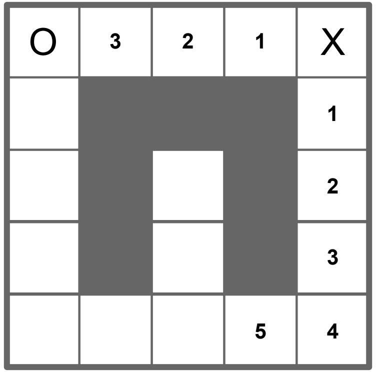


Mazes



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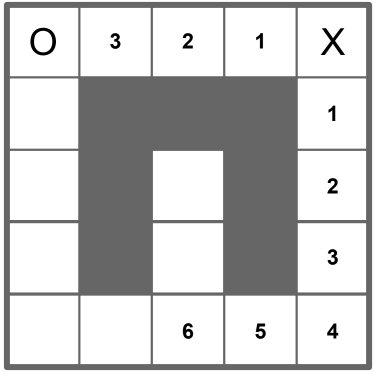

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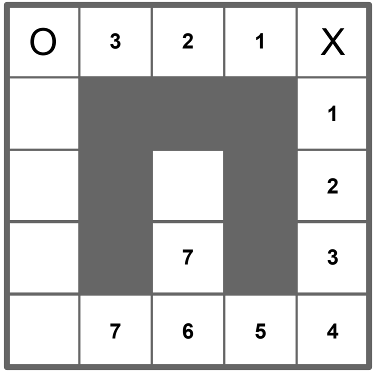

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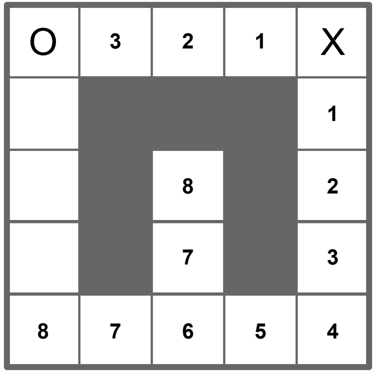

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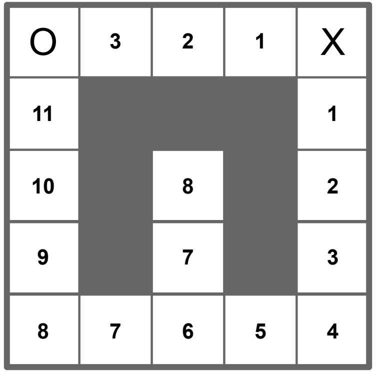


Mazes

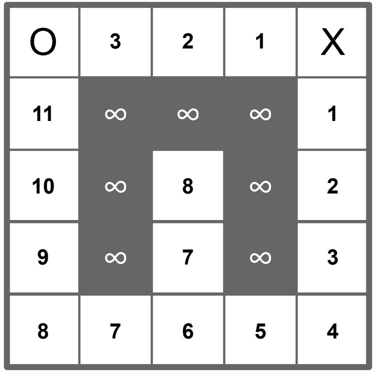
O	3	2	1	X
11	∞	∞	∞	1
10	∞	8	∞	2
9	∞	7	∞	3
8	7	6	5	4

Mazes

O	3	2	1	X
11	∞	∞	∞	1
10	∞	8	∞	2
9	∞	7	∞	3
8	7	6	5	4

So what is the number of steps from O to X?

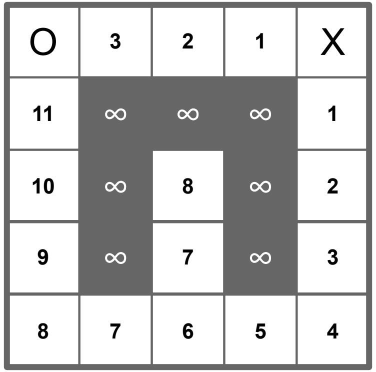
Mazes



So what is the number of steps from O to X?

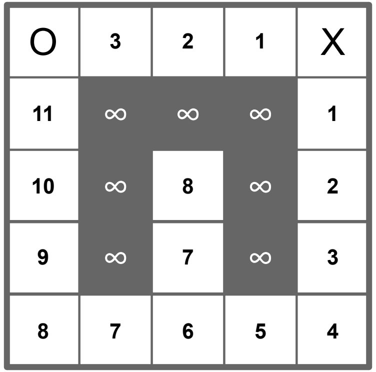
4 (min of neighbors + 1)

Mazes



Does this solution remind you of anything?

Mazes



Does this solution remind you of anything?

Recursion!

Mazes

$$steps(\texttt{pos}, \texttt{dest}) = \begin{cases} 0 & \textbf{if } \texttt{pos} = \texttt{dest} \\ \infty & \textbf{if } is\_filled(\texttt{pos}) \\ 1 + min\_adjacent(\texttt{pos}, \texttt{dest}) & \textbf{otherwise} \end{cases}$$

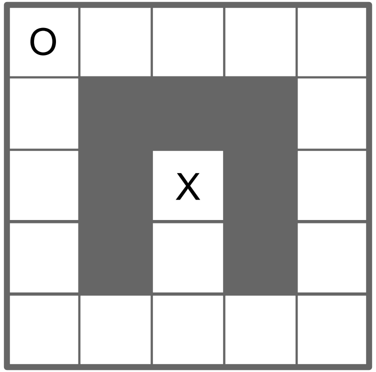
where...

$$min\_adjacent(\texttt{pos}, \texttt{dest}) = \min \begin{cases} steps(moveRight(\texttt{pos}), \texttt{dest}) \\ steps(moveDown(\texttt{pos}), \texttt{dest}) \\ steps(moveLeft(\texttt{pos}), \texttt{dest}) \\ steps(moveUp(\texttt{pos}), \texttt{dest}) \end{cases}$$

Mazes

```
steps(pos, dest):
  if pos == dest then return 0
  elif is_filled(pos) then return ∞
  else return 1 + min of
    steps(moveRight(pos, dest))
    steps(moveDown(pos, dest))
    steps(moveLeft(pos, dest))
    steps(moveUp(pos, dest))
```

Mazes



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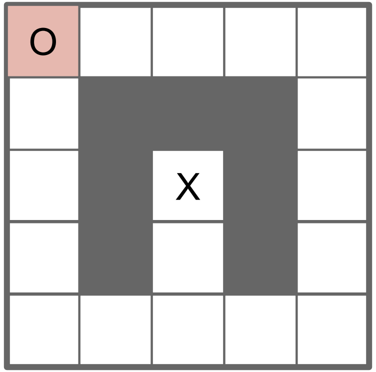

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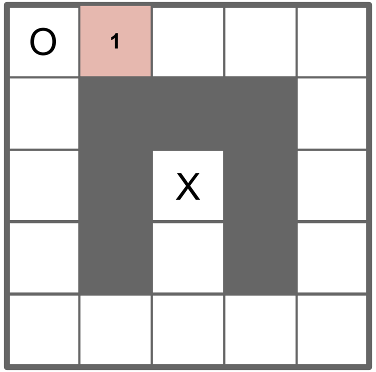

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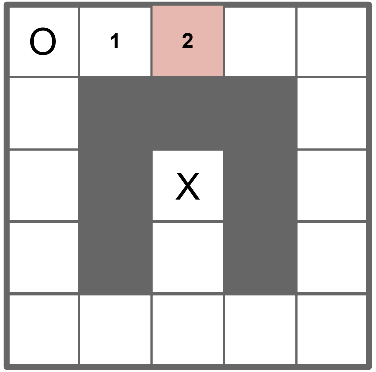

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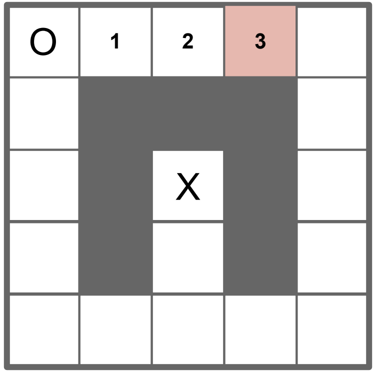

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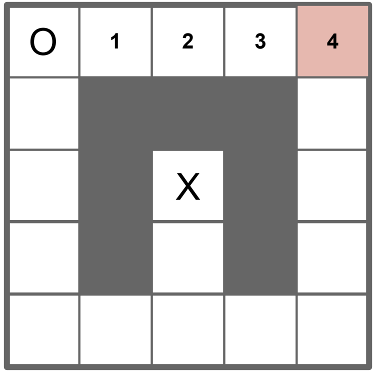

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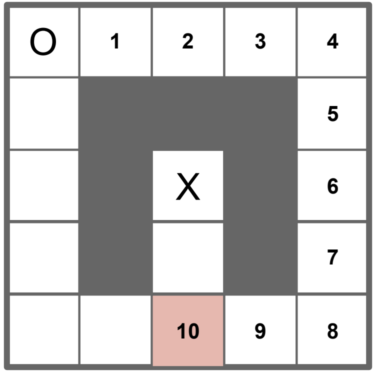

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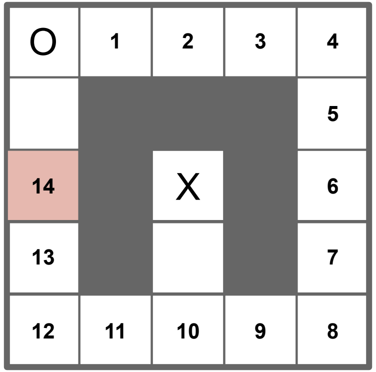

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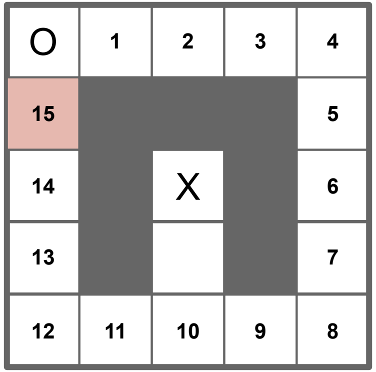

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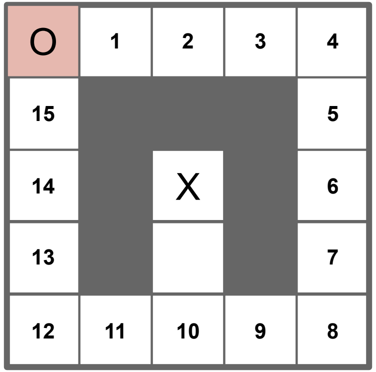

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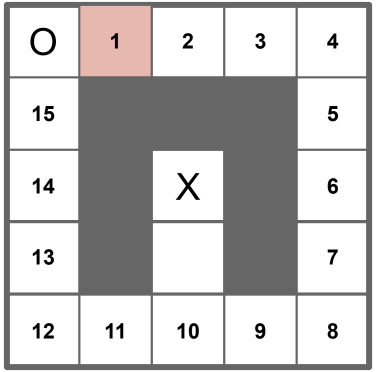

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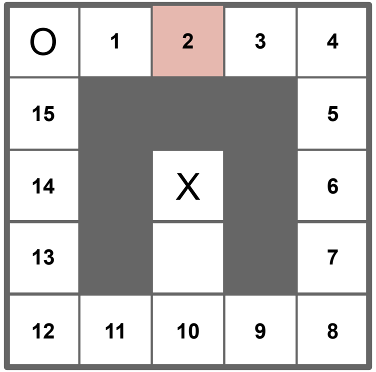

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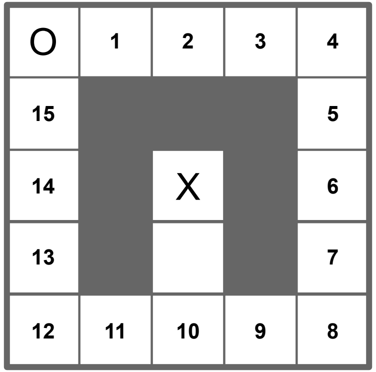
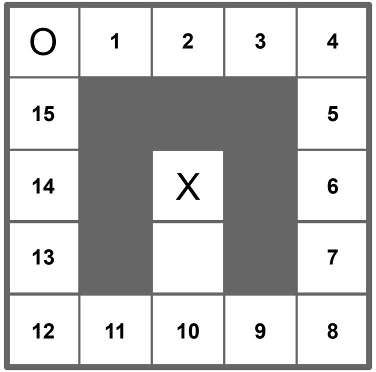


Problem: Infinite loop!

Mazes



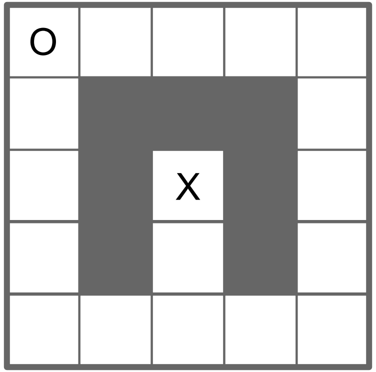
Problem: Infinite loop!

Insight: A path with a loop in it can't be shorter than one without the loop

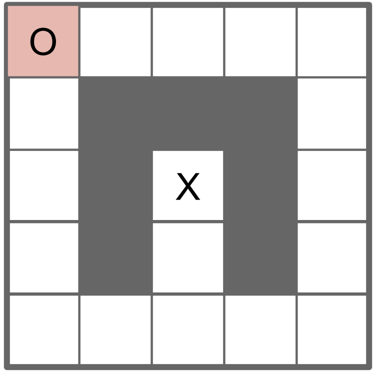
Mazes

```
steps(pos, dest):
  if pos == dest then return 0
  elif is_visited(pos) then return ∞
  elif is_filled(pos) then return ∞
  else
    Mark pos as visited
    return 1 + min of all 4 steps
```

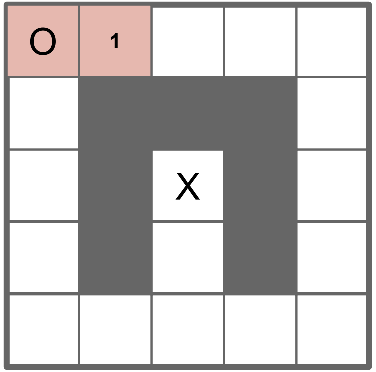
Mazes



Mazes



Mazes



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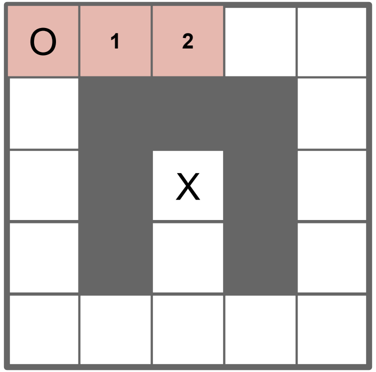

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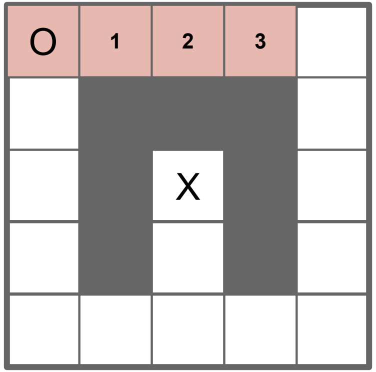
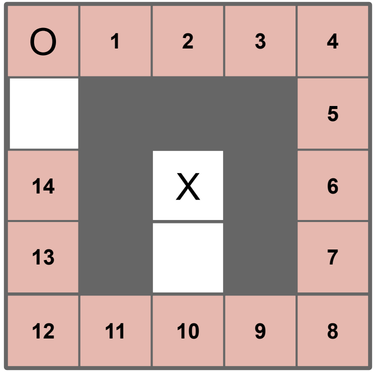


Mazes



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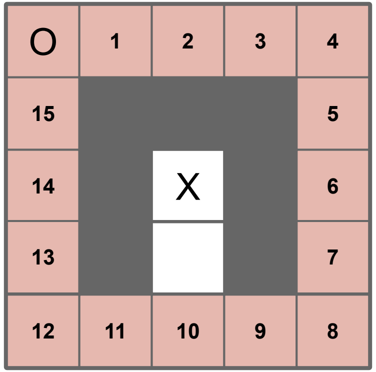
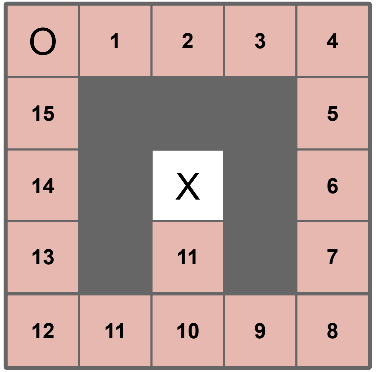


Mazes



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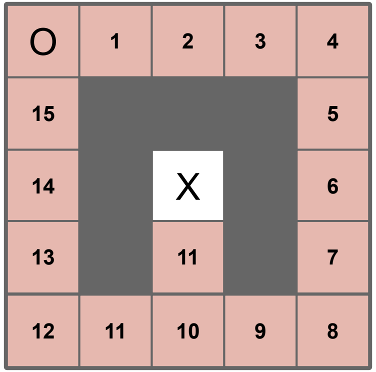
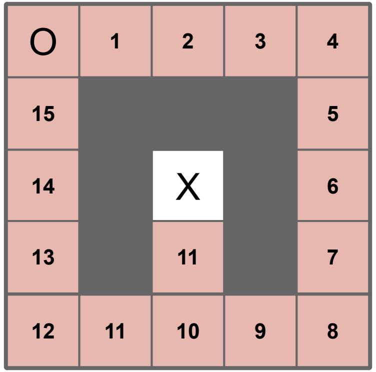


Problem: The first time you visit a node may be from a longer path!

Mazes



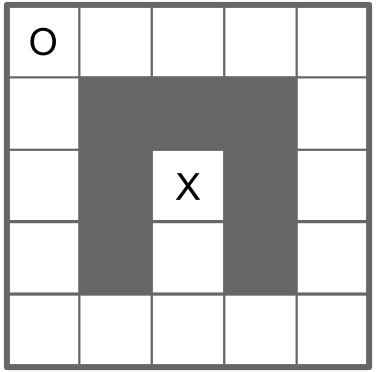
Problem: The first time you visit a node may be from a longer path!

Insight: Unmark nodes as you leave them

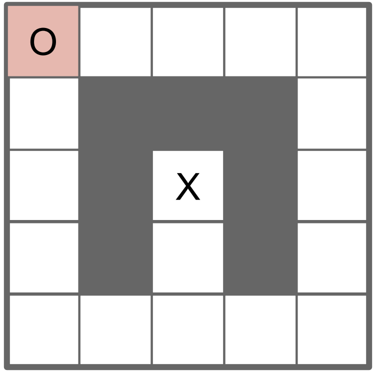
Mazes

```
steps(pos, dest):
  if pos == dest then return 0
  elif is_visited(pos) then return ∞
  elif is_filled(pos) then return ∞
  else
    Mark pos as visited
    min = 1 + min of all 4 steps
    Mark pos as unvisited
    return min
```

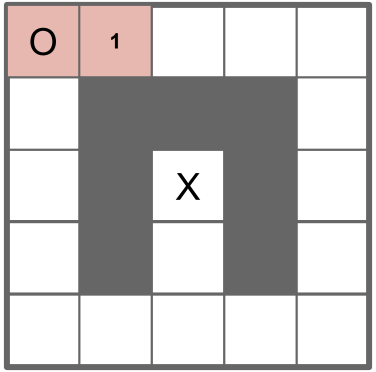
Mazes



Mazes



Mazes



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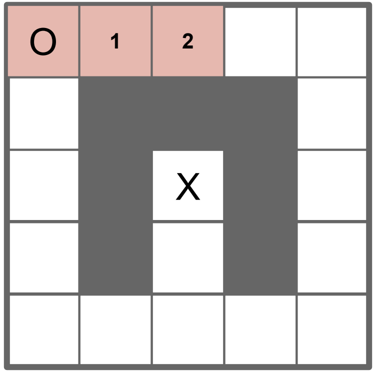

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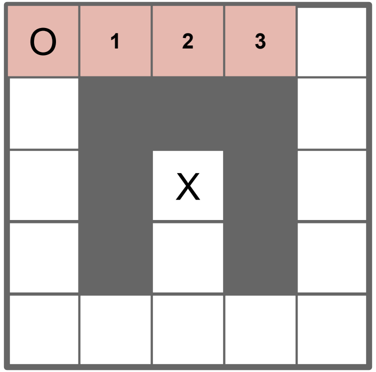

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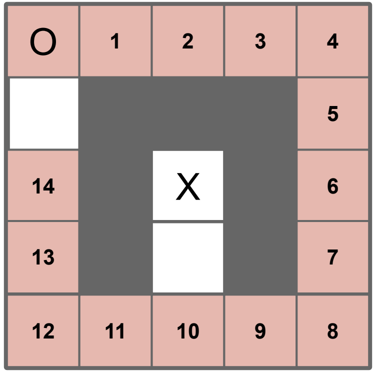

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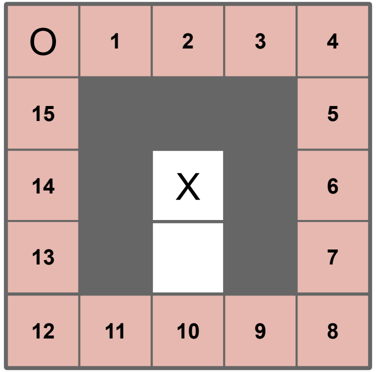

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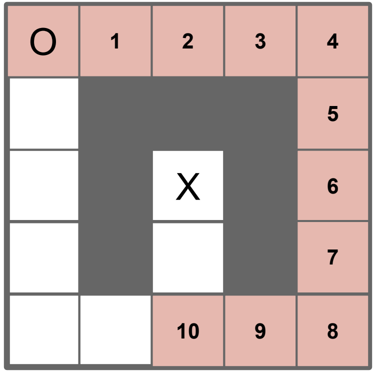

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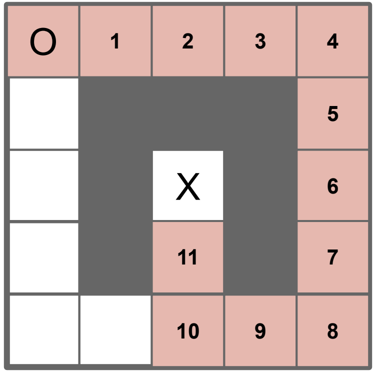

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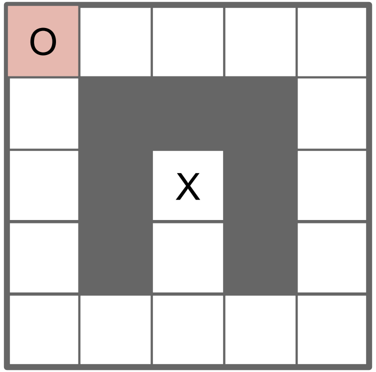

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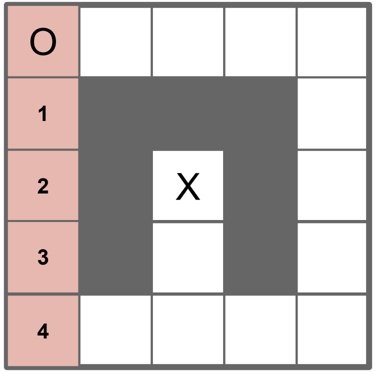

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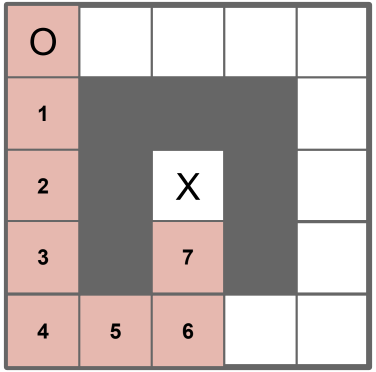

Formalizing Maze-Solving

Inputs:

- The map: an n x m grid of squares which are either filled or empty
- The **O** is at position *start*
- The **X** is at position *dest*

Goal: Compute `steps(start, dest)`, the minimum number of steps from start to end.

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Goal: Compute `steps(start, dest)`, the minimum number of steps from start to end. ✓

What path did we take?

Mazes

Idea: Keep track of the nodes marked visited…that's our path!

Mazes: Now with…some data structure?

```
steps(pos, dest, visited):
  if pos == dest then return visited.copy()
  elif pos ∈ visited then return no_path
  elif is_filled(pos) then return no_path
  else
    visited.append(pos)
    bestPath = 1 + min of all 4 steps
    visited.removeLast()
    return bestPath
```

Mazes: Now with...some data structure?

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    bestPath = 1 + min of all 4 steps
    visited.removeLast()
    return bestPath
```

What could this data structure be??

Mazes: Now with...Stacks!

```
steps(pos, dest, visited):
  if pos == dest then return visited.copy()
  elif pos ∈ visited then return no_path
  elif is_filled(pos) then return no_path
  else
    visited.push(pos)
    bestPath = 1 + min of all 4 steps
    visited.pop()
    return bestPath
```

A stack!

Queues?

Thought Experiment: Can we do something similar with queues?